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Uniform Spectral Amplitude Windowing for Hyperbolic Frequency Modulated Waveforms

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The hyperbolic frequency-modulated (HFM) waveform has been used in experimental and scientific low-frequency sonar systems because it possesses the pulse compression property and is Doppler-tolerant. This report presents a class of HFM waveforms that possess a flat spectrum, and as such is useful for the direct measurement of a target frequency response and the development of classification algorithms.

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CONTENTS

UNIFORM SPECTRAL AMPLITUDE WINDOWING FOR HYPERBOLIC-FREQUENCY MODULATED WAVEFORMS

INTRODUCTION

The development of classification algorithms within the Naval Research Laboratory Shallow Water Active Classification Project (ONR Project RJ14B87) requires that the response of the target be measured over a broad range of frequencies. This must be done with sufficient fidelity to resolve the subtle acoustic features of the target. It will allow algorithms to be developed that determine not only the class of target present (if one is present), but also target aspect.

A waveform used in an active sonar system that collects data for a classification algorithm must meet several criteria. First, the waveform must be broadband and fill all available bandspace offered by the transducer. The waveform should produce a high-resolution range profile of the target when matched filtered, so it must be a pulse compression waveform. Moreover, it should be Doppler-tolerant, that is, the zero-Doppler matched-filter response to a return should yield the range profile of the target even if the target is moving. Finally, the waveform should have a flat spectrum; this will allow a quick in situ look at the target spectrum without extensive post processing to compensate for the spectral coloring of the waveform. It will also simplify the design of a classifier.

With these criteria in mind, we consider the hyperbolic frequency-modulated (HFM) waveform as a good candidate. In its traditional form, it immediately meets most of these criteria; it is a broadband, Doppler-tolerant, pulse-compression waveform [1-5]. The waveform does not have a flat spectrum, especially below 1 kHz. However, with the correct time window, the spectrum can be made essentially flat. The design of such a window is the subject of this report.

THE HYPERBOLIC FREQUENCY-MODULATED WAVEFORM

The time-frequency function of the hyperbolic frequency-modulated waveform is given by

$$f(t) = \frac{f_1 f_2}{f_2 - Bt/T} \quad t \in [0, T], \tag{1}$$

where $f(0) = f_1$, $f(T) = f_2$, $B = f_2 - f_1$ is the bandwidth (in Hz), and T is the time length (in seconds) of the waveform. Note that Eq. (1) is a hyperbolic function, hence the name of the waveform.* If $f_1 < f_2$, then the waveform is an upsweep, otherwise, it is referred to as a downsweep. Waveforms using this form of time-frequency function possess the same pulse-compression property

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*Note that the waveform period given by $1 = f(t)$ is a linear function, thus, the waveform is also known as a linear period modulated (LPM) waveform. This name is commonly used among those working in high-frequency acoustics.

offered by the matched-filter processing of a linear frequency-modulated (LFM) waveform, but they are also Doppler-tolerant. That is, the matched-filter response to an echo from a moving point target will have a large peak value, even if the return is processed under the assumption that the target is stationary.

The functional form of the waveform with a rectangular window is given by

$$x(t) = \begin{cases} Ae^{\jmath\phi(t)} & \text{if } t \in [0, T], \\ 0 & \text{otherwise,} \end{cases} \tag{2}$$

where A is an arbitrary constant. From Eq. (1) the instantaneous phase is

$$\phi(t) = 2\pi \int_0^t f(\tau)d\tau = -\frac{2\pi f_1 f_2 T}{B} \ln\Big(f_2 T - Bt\Big). \tag{3}$$

Figure 1 shows the spectrum of an HFM waveform for $f_1 = 500$ Hz, $f_2 = 700$ Hz, and $T = 2$ seconds. Note the distinctive slant to the spectrum. This feature is most prominent in practical HFM waveforms whose spectral centroid is below 1 kHz. In the next section we derive a time window that will remove this feature of the spectrum.

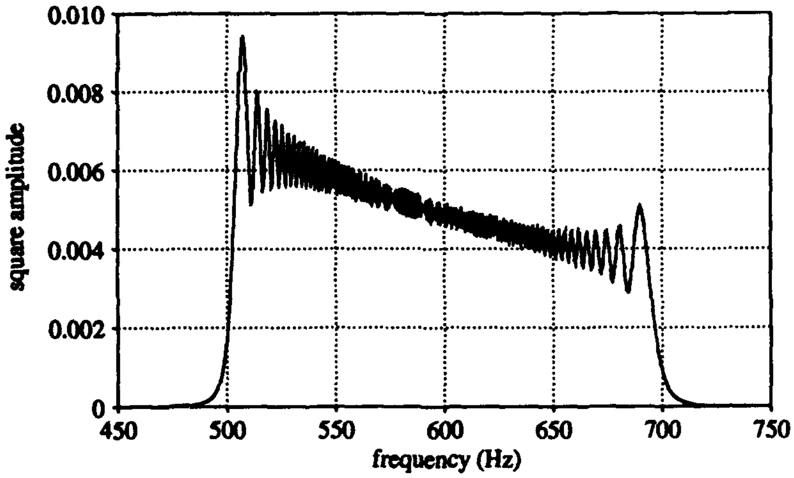

Fig. 1 – Spectrum of an HFM waveform with $f_1 = 500$ Hz, $f_2 = 700$ Hz, $T = 2$ seconds, and a rectangular window

WINDOW DESIGN FOR A FLAT SPECTRUM

Consider the case of an HFM waveform whose time-frequency function and phase are given by Eqs. (1) and (3), respectively, but whose time function is now given by

$$x(t) = \begin{cases} a(t)e^{\jmath\phi(t)} & \text{if } t \in [0, T], \\ 0 & \text{otherwise,} \end{cases} \tag{4}$$

where $a(t)$ is an arbitrary non-negative, real window. We now show that it is possible to chose $a(t)$ such that the waveform's spectrum will be flat. Here, the term 'flat' is used in a qualitative sense, since the example given below will show that the spectrum can still have a large amount of 'ripple.'

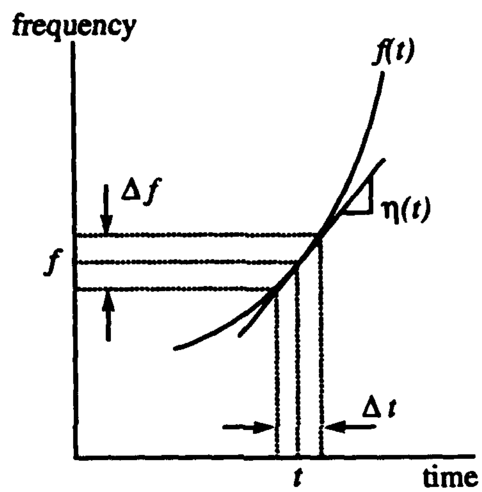

Fig. 2 – Time-frequency function of a frequency-modulated waveform, and the relationship between its slope, the dwell time Δt, and the frequency band Δf

Consider the arbitrary time-frequency function shown in Fig 2, denoted by $f(t)$. Also, let

$$\eta(t) \doteq df(t)/dt. \tag{5}$$

Consider a band of frequencies of size Δf centered about $f(t)$, then the approximate 'dwell time' of the waveform within this band is approximately given by

$$\Delta t = \frac{\Delta f}{|\eta(t)|}. \tag{6}$$

It follows that the approximate spectral energy in the band $\Delta f(t)$ is

$$E = a^2(t)\Delta t. \tag{7}$$

Since we want the spectrum to be flat, the amount of spectral energy in all bands of fixed size Δf should be constant, irrespective of our choice of t. Thus, from Eqs. (6) and (7) it follows that

$$a^2(t)\,\frac{\Delta f}{|\eta(t)|} = constant. \tag{8}$$

Equation (8) implies that our choice of the window requires

$$a(t) \propto \sqrt{|\eta(t)|}. \tag{9}$$

Note that this expression is not specific to an HFM waveform; it is valid for any frequency-modulated waveform. However, for the specific case of an HFM waveform, combining Eqs. (1), (5), and (9) yields

$$a(t) = \frac{K}{f_2 - Bt/T}, \tag{10}$$

where K is an arbitrary constant whose value depends on the application of the waveform. If the waveform is to be used in a peak-power-limited system, it must obey the property $0 \leq a(t) \leq 1$, implying that

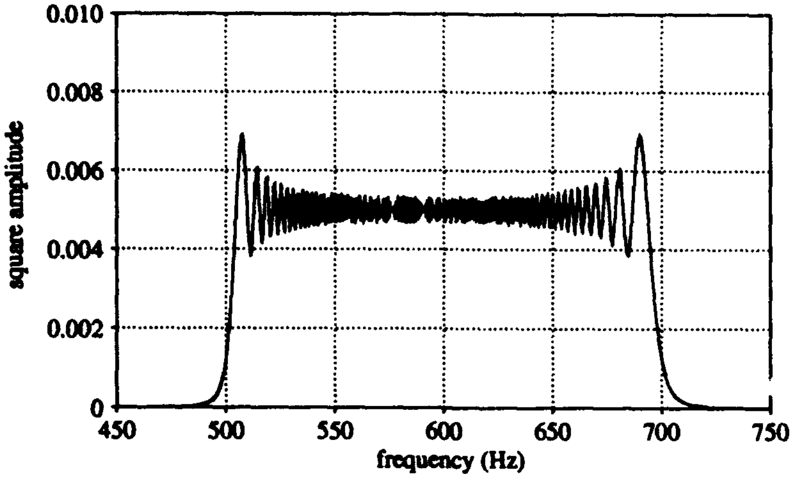

Fig. 3 – Spectrum of an HFM waveform with $f_1 = 500$ Hz, $f_2 = 700$ Hz, $T = 2$ seconds, and a window designed to remove spectral slant

$$a(t) = \frac{min(f_1, f_2)}{f_2 - Bt/T}. \tag{11}$$

If, however, the waveform is to be used for performing matched-filtering, then the energy normalized window can be found by integrating the square of the window given in Eq. (9), setting the result equal to 1, and solving for K. The result yields

$$a(t) = \frac{\sqrt{f_1 f_2 T}}{f_2 - Bt/T}. \tag{12}$$

Figure 3 shows the spectrum of an HFM waveform with the same frequencies and time length as the waveform whose spectrum is given in Fig. 1; however, we have applied the energy normalized window given in Eq. (12) and shown in Fig. 4. The spectrum is quite similar to that of a linear frequency-modulated (LFM) waveform.

REDUCTION OF SPECTRAL RIPPLE: COMPOSITE WINDOWING

Although the 'slant' to the spectrum of an HFM waveform can be removed by applying the window derived in the previous section, Fig. 3 shows that a large amount of spectral ripple can still be present. The ripple is also known as Gibb's phenomenon and is due to the presence of the jump discontinuities in the window at $t = 0$ and $t = T$ [6,7].

To reduce the spectral ripple, we propose the use of an additional window to make the waveform's functional definition continuous. In this report we use the tapered-cosine window given by

$$w(t) = \begin{cases} \frac{1}{2}\left[1 - \cos\left(\frac{\pi t}{\alpha T}\right)\right] & t \in [0, \alpha T), \\ 1 & t \in [\alpha T, (1-\alpha)T), \\ \frac{1}{2}\left[1 - \cos\left(\frac{\pi(T-t)}{\alpha T}\right)\right] & t \in [(1-\alpha)T, T], \\ 0 & \text{otherwise}, \end{cases} \tag{13}$$

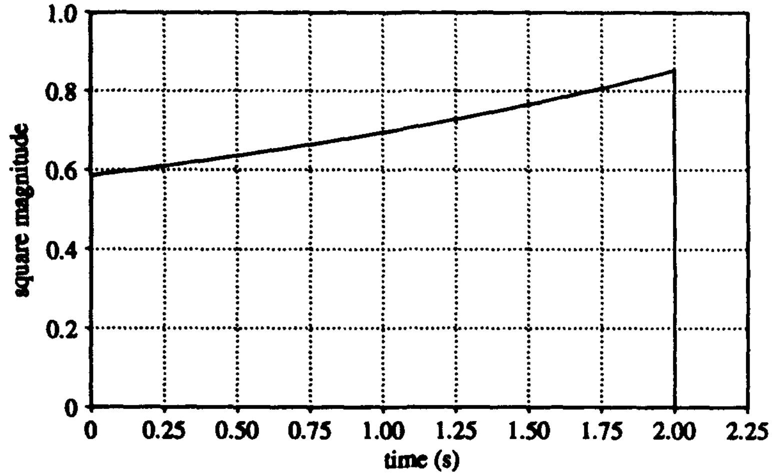

Fig. 4 – Envelope of an HFM waveform with $f_1 = 500$ Hz, $f_2 = 700$ Hz, $T = 2$ seconds, and a window designed to remove spectral slant

where α is a parameter that determines the effective width of the window, and $0 \le \alpha \le 0.5$. Use of the window in Eq. (13) implies that the waveform's functional form is now given by

$$x(t) = \begin{cases} w(t)a(t)e^{j\phi(t)} & \text{if } t \in [0,T], \\ 0 & \text{otherwise,} \end{cases} \tag{14}$$

where $a(t)$ is given by Eq. (10). There are, of course, other possible choices for $w(t)$, but the form in Eq. (13) was chosen because it is a common window type. It is also once continuously differentiable with a bounded second derivative, which implies that the waveform's spectrum obeys the property

$$X(\omega) = \int_0^T x(t)e^{j\omega t}dt \sim O\left(\frac{1}{|\omega - \omega_0|^3}\right) \quad \text{as } |\omega| \to \infty, \tag{15}$$

where ω_0 is the spectral centroid. This property implies that the 'spectral skirt' will be lower than that of the spectra for the waveforms given by Eqs. (2) and (4).

Figure 5 shows the spectrum of an HFM waveform with the same frequencies and time length as in Fig. 1. However, it has the functional form given in Eq. (14), for which the composite window $a(t)w(t)$ is shown in Fig. 6, with $\alpha = 0.1$. The waveform was energy normalized numerically. Clearly, the magnitude of the spectral ripple has been reduced.

If the waveform is to be used in a peak-power-limited system, then we must find the constant K such that $0 \le w(t)a(t) \le 1$, where $a(t)$ is given by Eq. (10). To do this, we first determine where the peak value of the window should occur. Some thought reveals that for an upsweep the peak should occur in the interval $t \in [(1-\alpha)T, T]$, and for a downsweep the peak should occur in the interval $t \in [0, \alpha T]$. Furthermore, the derivative of the composite window must be equal to zero at the peak. Hence, differentiating $a(t)w(t)$, and setting the result equal to zero leads to the following equation whose solution gives the location of the peak if the waveform is an upsweep ($f_1 < f_2$):

$$B\alpha\left\{1 - \cos\left[\frac{\pi}{\alpha}\left(1 - \frac{t}{T}\right)\right]\right\} - \pi\left(f_2 - \frac{Bt}{T}\right)\sin\left[\frac{\pi}{\alpha}\left(1 - \frac{t}{T}\right)\right] = 0 \quad \text{for } t \in [(1-\alpha)T, T]. \tag{16}$$

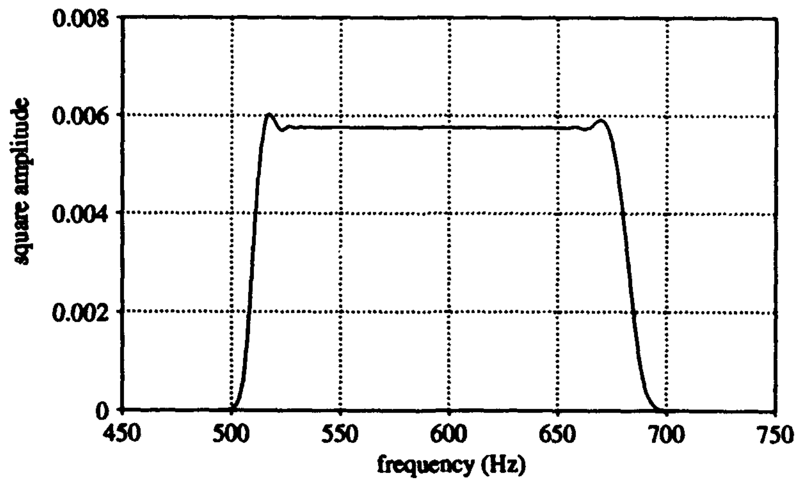

Fig. 5 – Spectrum of an HFM waveform with $f_1 = 500$ Hz, $f_2 = 700$ Hz, $T = 2$ seconds, and a composite window with $\alpha = 0.1$

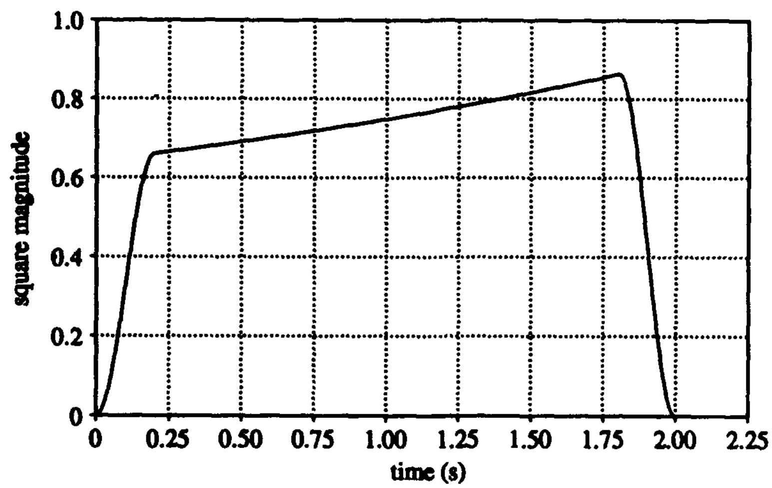

Fig. 6 – Envelope of an HFM waveform with $f_1 = 500$ Hz, $f_2 = 700$ Hz, $T = 2$ seconds, and a composite window with $\alpha = 0.1$

By a similar procedure, it is possible to derive the following equation whose solution gives the location of the peak if the waveform is a downsweep ($f_1 > f_2$):

$$B\alpha\left[1-\cos\left(\frac{\pi t}{\alpha T}\right)\right]+\pi\left(f_2-\frac{Bt}{T}\right)\sin\left(\frac{\pi t}{\alpha T}\right)=0 \quad \text{for } t \in [0, \alpha T]. \tag{17}$$

Denoting the solution to either Eq. (16) or (17) as t_p, it follows from (10) that $K = f_2 - Bt_p/T$.

Obviously the equations in (16) and (17) are nonlinear and must be solved numerically. However, several numerical algorithms can be used to find the solution, such as the bisection method, secant method, regula falsa, and Newton-Raphson. Luckily, some relief from this problem can be found by using mathematical software packages such as *Mathematica* [8] or *Maple* [9] that have these numerical algorithms preprogrammed as functions.

BANDWIDTH COMPENSATION

Use of the tapered-cosine window will trim the leading and trailing edges of the envelope, and consequently remove those spectral components associated with the beginning and end of the pulse. The result is a reduction of the true bandwidth of the waveform; this can be seen by comparing the spectra shown in Figs. 3 and 5. In this case, the window effectively removed the leading and trailing 5% of the energy in the window, hence, the spectrum in Fig. 5 is approximately 10% narrower than the spectrum shown in Fig. 3.

To design a waveform with the desired bandwidth B, we must adjust the values of the frequencies f_1 and f_2 in Eq. (1). In this case, we define the 'design frequencies' $\hat{f}_1$ and $\hat{f}_2$ such that $B = \hat{f}_2 - \hat{f}_1$, and now refer to f_1 and f_2 as the 'parameter frequencies.' The design frequencies are known. The parameter frequencies are used in Eqs. (1) and (3) and must be derived from the design frequencies.

To derive design equations for the parameter frequencies we make the following observation: the tapered-cosine window as given in Eq. (13) effectively trims the leading and trailing $100(\alpha/2)\%$ of the energy from the rectangular window, so the bandwidth is approximately $100\alpha\%$ of that for the rectangularly windowed HFM. To compensate for this effect, we set $\hat{f}_1 = f(\alpha T/2)$ and $\hat{f}_2 = f((1-\alpha/2)T)$. Thus, from Eq. (1) and the definition of the bandwidth B, it is possible to show that

$$\begin{aligned} \frac{1-\alpha/2}{f_1} + \frac{\alpha/2}{f_2} &= \frac{1}{\hat{f}_1}, \\ \frac{\alpha/2}{f_1} + \frac{1-\alpha/2}{f_2} &= \frac{1}{\hat{f}_2}. \end{aligned} \tag{18}$$

The simultaneous equations in (18) can be easily solved for the unknown values f_1 and f_2 by using Gaussian elimination, or the method of determinants. The solution is

$$\begin{aligned} f_1 &= \frac{2(1-\alpha)}{(2-\alpha)/\hat{f}_1 - \alpha/\hat{f}_2}, \\ f_2 &= \frac{2(1-\alpha)}{(2-\alpha)/\hat{f}_2 - \alpha/\hat{f}_1}. \end{aligned} \tag{19}$$

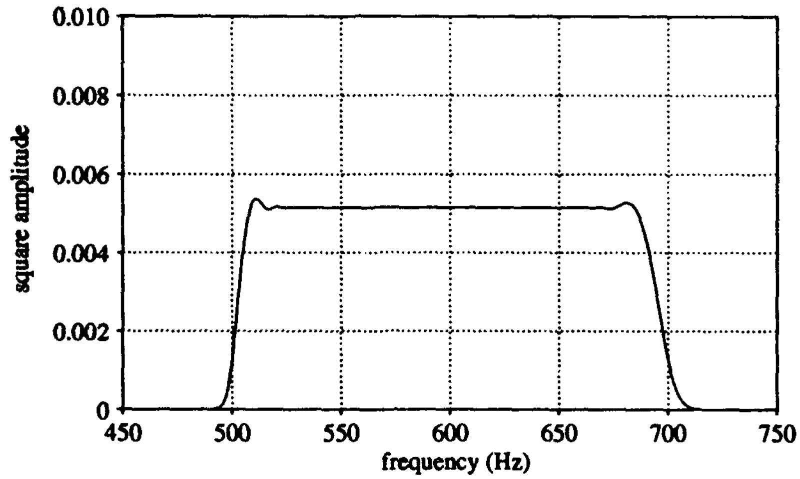

Fig. 7 – Spectrum of an HFM waveform with $\hat{f}_1 = 500$ Hz, $\hat{f}_2 = 700$ Hz, $T = 2$ seconds, and a composite window with $\alpha = 0.1$

We continue with the examples used in the previous sections based on an HFM waveform with $\hat{f}_1 = 500$ Hz, $\hat{f}_2 = 700$ Hz, and $\alpha = 0.1$. Substituting these values of the design frequencies into the equations in (19) yields $f_1 = 492.19$ Hz and $f_2 = 715.91$ Hz. Figure 7 shows the spectrum of a unit energy waveform using these parameter frequencies. It can be seen that it has the same bandwidth as the waveform that uses a rectangular window, whose spectrum is shown Fig. 3.

The autocorrelation function of the waveform $x(t)$ is defined as

$$R(\tau) = \left| \int_{-\infty}^{\infty} x(t)x^*(t-\tau)dt \right|^2; \tag{20}$$

Fig. 8 shows the autocorrelation functions of two of the HFM waveforms given in the examples. The dashed line is the autocorrelation of the rectangularly windowed HFM whose spectrum is shown in Fig. 1. The solid line is the autocorrelation function of the HFM presented in this section, to which we have applied composite windowing and bandwidth compensation. Note that the shape of the main peak and the location of the sidelobes are essentially the same for both waveforms. This demonstrates that the use of the composite window and bandwidth compensation can have negligible effects on the resolution of the waveform.

CONCLUSIONS

We have presented a method of designing HFM waveforms that have essentially flat spectra possessing a small amount of ripple. This was accomplished by selecting an appropriate time window. The form of the window $a(t)$ given in Eq. (10) was also suggested by Kroszczyński in Ref. 4, but its derivation is contained in a reference cited in Ref. 10. It is not known if the derivation of the window is similar to the approach presented here, since the reference is not available.

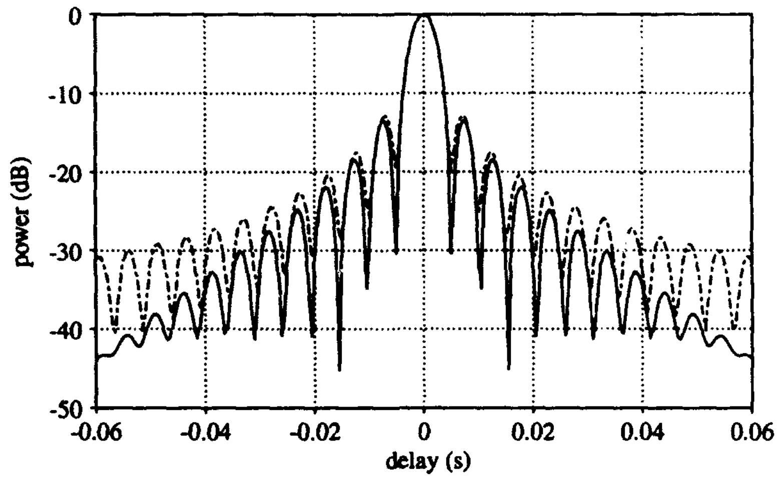

Fig. 8 - Autocorrelation functions of two HFM waveforms. Dashed line is the autocorrelation of an HFM waveform with $\hat{f}_1 = 500$ Hz, $\hat{f}_2 = 700$ Hz, $T = 2$ seconds, and a rectangular window. Solid line is an HFM with the same design frequencies and time length but with a composite window ($\alpha = 0.1$) and bandwidth compensation.

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